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OPTIMAL CARBON TAXATION WITH CARBON LEAKAGE AT THE EXTENSIVE AND THE INTENSIVE MARGIN

Peter Birch Sørensen

Abstract: We analyse the optimal design of carbon taxation in a small open economy where the government is committed to a target for reduction of CO_2 emissions from domestic territory but where it is also concerned about carbon leakage. We highlight the importance of distinguishing between between leakage at the extensive margin where firms may relocate to a foreign country to avoid the domestic carbon tax, and leakage at the intensive margin where domestic firms lose world market shares to foreign competitors due to the tax. Our analysis shows that when leakage occurs at the extensive margin, the leakage rate is likely to be much higher than the rate of leakage at the intensive margin. The optimal carbon tax scheme therefore includes a lump sum subsidy to domestic firms to counteract carbon leakage at the extensive margin. The subsidy may be implemented by taxing emissions above a historical baseline level and subsidizing emission reductions below the baseline level at a similar rate. Optimal taxation also requires that carbon tax rates be differentiated across sectors according to their different leakage rates.

Keywords: Carbon leakage, optimal carbon taxation in an open economy.
JEL codes: H21, H23, Q48, Q54.

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1. The problem

Hoping to lead the world community by example, several European countries have adopted ambitious targets for greenhouse gas emission reductions that go well beyond their obligations towards the European Union. As Hoel (2012) and Greaker et al. (2019) explain, small countries may affect emission reduction programs in other countries in a number of ways, e.g., via demonstration effects and development of green technologies. However, a well-known problem facing frontrunners in climate policy is that domestic emissions may simply leak abroad to countries that have not yet committed themselves to binding targets for emission reductions. Although the rate of carbon leakage may be less than 100 percent, it can still nullify a large part of the global impact of the climate policy effort of frontrunner countries.

To prevent carbon leakage, several academics and policy makers have suggested that countries with ambitious climate policies could introduce a system of border tax adjustments, imposing a tax on the estimated carbon content of imported goods and offering a rebate for the domestic carbon tax on the production of exported goods. Under such a system, analyzed by Hoel (1996), Böhringer et al. (2012), and Fischer and Fox (2012), among others, the competitiveness of domestic vis á vis foreign producers could in principle be preserved, at least if the system does not trigger a trade war with the country's trading partners.

The idea of introducing some form of border carbon tax adjustment was recently revived in the "European Green Deal" proposed by the European Commission (2019). However, such a border adjustment would have to be based on an average carbon price agreed upon by EU member

[1] This paper grew out of discussions with Kristian Binderup, Jens Hauch, Peter Kjær Kruse-Andersen, Hans Jørgen Whitta-Jacobsen and several other colleagues at the University of Copenhagen and the Kraka think tank. I am grateful for inspiration from all of these colleagues without implicating them in any shortcomings of the paper.

states, so it would not be sufficient to prevent carbon leakage to non-EU countries from member states adopting a more ambitious climate policy than the average EU policy stance. Moreover, it remains to be seen whether member states can agree on a system of border carbon adjustment and whether such a system can be implemented without provoking a counterproductive retaliation from major EU trading partners.

Against this background, the present paper proposes an alternative (or complementary) carbon tax scheme that could prevent most of the potential carbon leakage from countries that want to inspire the rest of the world by moving ahead in climate policy. The tax scheme involves a carbon tax on emissions above a baseline level with a similar subsidy to reductions of emissions below the baseline. As shown in the next section, the system works like an ordinary carbon tax combined with a lump sum subsidy to mobile firms locating production within the domestic economy. With an appropriate choice of the baseline emission level, the scheme can prevent most of the carbon leakage that would otherwise occur through a relocation of firms to foreign countries.

To my knowledge, this paper is the first one to highlight the importance of distinguishing between carbon leakage at the intensive and the extensive margin when designing an optimal carbon tax scheme. At the intensive margin emissions leak abroad as domestic firms react to a cost-increasing domestic carbon tax by reducing their output, thereby losing world market shares to competing foreign firms. At the extensive margin emissions leak from the domestic to the foreign economy as firms relocate their production plants to foreign jurisdictions. The analysis below indicates that, when relocation occurs, the carbon leakage rate will be far greater than the leakage rate at the intensive margin and that the proposed carbon tax scheme reduces the welfare loss from leakage.

In line with the 2015 Paris Agreement according to which countries are responsible for greenhouse gas emissions from their own territory, my analysis assumes that the home country is committed to a binding target for reducing domestic emissions. The domestic social planner wishes to meet this target at a minimum welfare cost to the representative domestic citizen. This welfare cost includes a welfare loss from carbon leakage, reflecting that citizens do not only care about their country's formal international climate policy obligations, but also about the net impact of domestic policy on the global climate. One real-world example illustrating the relevance of this assumption is the recent Danish Climate Act which commits the Danish government to reducing domestic greenhouse gas emissions by 70 percent in 2030 relative to the 1990 emission level, but to do so in a way which ensures that "..Danish policy measures do not simply move all of the emissions beyond Danish borders", as stated in the preamble of the Act.

The paper derives formulas for the optimal values of the parameters of the carbon tax scheme sketched above, showing how the optimal national climate policy depends on key parameters such as the estimated rates of carbon leakage from the household sector and from the business sector at the extensive and the intensive margins. The paper complements the one by Kruse-Andersen and Sørensen (2019) who study optimal energy taxes and subsidies in a small economy exposed to carbon leakage where the government is committed to a target for the country's contribution to *global* emissions. Here I focus on a situation where the domestic government wishes to demonstrate to the rest of the world how to meet an ambitious target for reduction of *territorial* emissions, as required by the Paris Agreement, but where the government is also concerned about carbon leakage. Further, I show how the existence of carbon leakage at the extensive margin calls for a climate policy instrument targeted directly at influencing firms' location decisions.

The carbon tax scheme described here may be seen as an alternative to a system with border carbon adjustment (BCA), or as a complement (for ambitious frontrunner countries) to a possible future EU-wide BCA based on a minimum carbon price. As discussed by Böhringer et al. (2012) and Böhringer et al. (2017), there are other ways of counteracting carbon leakage such as industry exemptions from carbon regulation and output-based allocation of emission allowances, possibly combined with consumption taxes on emission-intensive trade-exposed goods. One attractive feature of the carbon tax scheme proposed here is that it is relatively simple and allows policy makers to address concerns about equity (across production sectors) as well as efficiency, as explained in the next section.

The paper is organized as follows. Section 2 lays out the carbon tax scheme to be analyzed. Section 3 provides numerical examples based on a simple partial equilibrium model to illustrate the importance of distinguishing between carbon leakage at the extensive and the intensive margin. Section 4 sets up a simple general equilibrium model of a small open economy as a basis for analyzing the optimal design of carbon taxation in the presence of both types of leakage. Section 5 uses the model to derive formulas for the optimal carbon tax parameters and considers optimal taxation in some interesting benchmark cases. The final section 6 sums up my conclusions, and two technical appendices document some results reported in the main body of the paper.

2. A carbon tax scheme addressing leakage

Under the carbon tax scheme analyzed in this paper, a firm's carbon tax liability *B* would be given by the formula

$$B = \tau_b \left(e - \overline{e} \right), \qquad \tau_b > 0, \quad \overline{e} > 0, \tag{1}$$

where τ_b is the tax per unit of CO_2 emissions, e is the current emission level, and $\overline{e}$ is a baseline emission level exogenous to the firm. According to (1) the net tax liability could be either positive or negative: if the firm reduces emissions below the baseline level, it receives a subsidy τ_b for each unit of emission reduction below the baseline, so the firm faces a constant opportunity cost equal to τ_b per unit of carbon emitted regardless of its emission level.

The baseline emission level $\overline{e}$ could be set as a fraction of the firm's average historical emissions (or on the basis of its average recorded fossil fuel use) over some previous period. This would automatically account for inherent differences in energy intensity across firms and would ensure that highly energy intensive firms and sectors would not have to bear a disproportionate part of the total tax burden, even though they would face the same carbon price τ_b at the margin as all other firms. Choosing an appropriate rule for calculating the baseline emission level based on historical records allows the government to address concerns about equity across different production sectors without enabling firms to manipulate the basis for setting the baseline level.[2] Since the carbon tax would gradually reduce the carbon intensity of production, the baseline emission level could be slowly reduced over time according to a fixed schedule; the important thing is that individual firms should not be able to influence their own baseline level.

The tax scheme (1) is obviously equivalent to a standard carbon tax combined with a lump sum subsidy, since (1) implies

$$B = \tau_b e - T_b, \qquad T_b = \tau_b \bar{e}. \tag{2}$$

An internationally mobile firm can avoid the domestic carbon tax by relocating production to a foreign country, but then it would also lose the subsidy $T_b = \tau_b \bar{e}$. Thus the subsidy only has a lump sum character as long as the firm stays at home; essentially the baseline emission level provides a disincentive to relocate abroad and thus helps to prevent carbon leakage at the extensive margin. At the intensive margin leakage will still occur, since the carbon tax τ_b raises the marginal cost of domestic production, thereby hurting the international competitiveness of domestic firms. But as the next section will show, carbon leakage at the intensive margin is likely to be much smaller than the potential leakage at the extensive margin.

3. Carbon leakage at the extensive and the intensive margin: a partial equilibrium analysis

To make this point I use a parsimonious model of a firm whose output *x* is given by the concave production function

$$x = f(e), \qquad f' > 0, \quad f'' < 0, \tag{3}$$

where *e* is the input of fossil fuel. We choose units such that the burning of one unit of fossil fuel generates one unit of CO_2 emissions. The production function $f(e)$ also includes an input factor which is fixed once the firm's location decision has been made, but which may be moved abroad

[2] For new firms the baseline emission level could be set on the basis of some benchmark historical emission level for the industry concerned.

by incurring a fixed relocation cost. The firm's output is traded internationally at the world market price *P* determined endogenously in the way described below, and fossil fuel is traded at the exogenous producer price *p*. If the firm locates at home, it will be subject to the carbon tax scheme (2), so the profit π accruing to the owner of the fixed factor will be

$$\pi = Pf(e) - (p + \tau_b)e + T_b. \tag{4}$$

Under perfect competition the firm's maximum attainable profit from domestic production is given by a profit function of the form

$$\pi = \pi(P, p + \tau_b) + T_b, \qquad \frac{\partial \pi}{\partial P} = f(e), \qquad \frac{\partial \pi}{\partial (p + \tau_b)} = -e, \tag{5}$$

where e is the firm's demand for fossil fuel derived from the condition for profit maximization

$$f'(e) = \frac{p + \tau_b}{P}. \tag{6}$$

If the firm moves the fixed factor to a foreign location with no carbon tax, its maximum attainable profit from foreign production will be $\pi(P, p) > \pi(P, p + \tau_b) + T_b$, but it will then have to incur a firm-specific fixed relocation cost of *c* (measured in annuity terms). The firm will therefore prefer to stay at home as long as

$$\pi(P, p) - \pi(P, p + \tau_b) - T_b \leq c. \tag{7}$$

Condition (7) will play a key role in the general equilibrium analysis in sections 4 and 5. For the moment, we will investigate what difference it makes for global CO_2 emissions whether the firm moves abroad rather than staying at home. For the purpose of numerical analysis, suppose the production function (3) takes the form

$$f(e) = Ae^{\beta}, \qquad 0 < \beta < 1, \tag{8}$$

where *A* is the fixed factor that may be moved abroad. With asterisks indicating activities occurring in a foreign location, we then obtain the following results from the condition for profit maximization:

$$\text{Domestic location:} \qquad e=\left(\frac{P\beta A}{p+\tau_b}\right)^{\frac{1}{1-\beta}}, \qquad x=A^{\frac{1}{1-\beta}}\left(\frac{P\beta}{p+\tau_b}\right)^{\frac{\beta}{1-\beta}}, \tag{9}$$

$$\text{Foreign location:} \qquad e^*=\left(\frac{P\beta A}{p}\right)^{\frac{1}{1-\beta}}, \qquad x^*=A^{\frac{1}{1-\beta}}\left(\frac{P\beta}{p}\right)^{\frac{\beta}{1-\beta}}. \tag{10}$$

Suppose the total world demand X^d for the output of the industry considered is

$$X^d = aP^{-\varepsilon}, \qquad a>0, \qquad \varepsilon>0, \tag{11}$$

where a is a constant scale factor, and ε is the constant numerical price elasticity of demand. Suppose further that the domestic firm faces a total of n foreign competitors in the international market, all using the same production technology as its own technology. The output and emissions of these foreign firms will then be given by (10) which also describes the output and emissions of the domestically owned firm in case it moves abroad. We now have the following conditions for market equilibrium depending on whether the domestically owned firm stays at home or relocates abroad:

$$\text{Market equilibrium without relocation:} \qquad x+nx^*=X^d, \tag{12}$$

$$\text{Market equilibrium with relocation:} \qquad (1+n)x^*=X^d. \tag{13}$$

The left-hand sides of (12) and (13) are the total world supply of the good considered, calculated from (9) and (10). Using (9) through (13), Appendix A derives the equilibrium output price P and the resulting emissions from the domestic firm as well as total global emissions. Denoting global emissions by E^g and the ad valorem carbon tax rate by t, we obtain the following results in the two location scenarios:

Emissions without relocation:

$$e=\left(\frac{1}{1+t}\right)^{\frac{1}{1-\beta}}\left\{\frac{a}{A}\left(\frac{\beta A}{p}\right)^{\varepsilon}\frac{1}{(1+t)^{\frac{-\beta}{1-\beta}}+n}\right\}^{\frac{1}{\varepsilon(1-\beta)+\beta}}, \qquad t\equiv\frac{\tau_b}{p}, \tag{14}$$

$$E^g=\left\{\frac{(1+t)^{\frac{-1}{1-\beta}}+n}{\left[(1+t)^{\frac{-\beta}{1-\beta}}+n\right]^{\frac{1}{\varepsilon(1-\beta)+\beta}}}\right\}\left\{\frac{a}{A}\left(\frac{\beta A}{p}\right)^{\varepsilon}\right\}^{\frac{1}{\varepsilon(1-\beta)+\beta}}, \tag{15}$$

Emissions with relocation (= initial emissions)

$$e=e^*=\left\{\frac{a}{A}\left(\frac{\beta A}{p}\right)^{\varepsilon}\frac{1}{1+n}\right\}^{\frac{1}{\varepsilon(1-\beta)+\beta}}, \tag{16}$$

$$E^g=(1+n)^{\frac{(\varepsilon-1)(1-\beta)}{\varepsilon(1-\beta)+\beta}}\left\{\frac{a}{A}\left(\frac{\beta A}{p}\right)^{\varepsilon}\right\}^{\frac{1}{\varepsilon(1-\beta)+\beta}}. \tag{17}$$

Note that when the domestic firm moves all of its activity abroad, its own emissions as well as global emissions are the same as before the domestic carbon tax scheme is introduced, since the firm uses the same technology and faces the same energy price as before, leaving total world supply of the final good (and hence its equilibrium price) unchanged. Thus the rate of carbon leakage at the extensive margin is exactly 100 percent, as the emissions from foreign territory increase by the same amount as the fall in domestic emissions.

When the lump sum subsidy T_b is sufficient to motivate the firm to stay at home, the fall in its emissions caused by the introduction of the carbon tax may be found by subtracting (14) from (16), and the fall in global emissions induced by the tax can be derived by subtracting (15) from (17). Since the tax causes the domestic firm to reduce its output, the equilibrium output price increases, inducing foreign firms to increase their output and emissions. In this way some of the previous domestic emissions will leak abroad as foreign firms increase their market share at the expense of the domestic firm. The rate of this carbon leakage via the intensive margin is given by the formula

$$\text{Rate of carbon leakage} = 1-\Omega, \qquad \Omega = \frac{\text{Fall in global emissions}}{\text{Fall in domestic emissions}}. \tag{18}$$

In Table 1 I have used equations (14) through (18) to calculate the rate of carbon leakage at the intensive margin for alternative parameter values. The parameter $\theta \equiv 1/\left(1+n\right)$ is the world market share of the domestic firm in the initial equilibrium before the carbon tax is introduced. The domestic firm represents the entire domestic industry in the market considered, so even if individual competitive firms are very small relative to the world market, the domestic industry as a whole may have a non-negligible market share.

In all of the cases considered in Table 1, I assume that the government sets the carbon tax rate $t \equiv \tau_b / p$ so as to induce a 50 percent fall in domestic emissions from the industry considered, measured relative to the initial no-tax equilibrium. Depending on parameter values, this policy target will require slightly different tax rates, as indicated in the upper row of the table. The benchmark scenario in the first column assumes that the domestic industry has an initial world market share of 5 percent, that the numerical price elasticity of world demand for the final good is 1 (Cobb-Douglas preferences), and that the industry considered is fairly energy-intensive with the energy bill comprising 10 percent of the value of gross output.

Table 1. Rate of carbon leakage at the intensive margin.

	Benchmark[1]	$\theta = 0.01$	$\theta = 0.1$	$\beta = 0.05$	$\beta = 0.2$	$\varepsilon = 0.5$	$\varepsilon = 5$
Carbon tax rate (t)[2]	0.872	0.867	0.878	0.935	0.750	0.877	0.868
Leakage rate (1-Ω)	0.128	0.133	0.123	0.065	0.250	0.234	0.028

1. Benchmark calibration: $\theta = 0.05$, $\beta = 0.1$, $\varepsilon = 1$.
2. In all scenarios the carbon tax rate is set so as to induce a 50 percent fall in domestic emissions.

Source: Own calculations based on equations (14) through (18).

Table 1 shows that the rate of carbon leakage at the intensive margin is not very sensitive to the size of the market share of domestic firms, but not surprisingly, the leakage rate increases significantly in case of a substantial increase in the energy intensity of production (β). The leakage rate is also rather sensitive to the price elasticity of demand for the final good. When the price elasticity is low, the tax-induced fall in domestic supply to the world market requires a large price increase to clear the market which in turn induces a relatively large increase in output and emissions from foreign firms, thereby offsetting a significant part of the fall in domestic emissions. However, in all of the scenarios in Table 1 the leakage rate at the intensive margin is much lower than the 100 percent leakage occurring at the extensive margin if domestic firms relocate to foreign jurisdictions.

The estimated leakage rates in Table 1 are based on a partial equilibrium analysis. To capture the impact on leakage stemming from the interaction between different sectors, one would have to use a general equilibrium model of the entire world economy. However, it seems unlikely that a general equilibrium analysis would overturn the conclusion from the numerical analysis above that carbon leakage at the extensive margin, when it occurs, will typically be much larger than leakage at the intensive margin.[3] This suggests that a carbon tax scheme which includes an instrument to offset leakage at the extensive margin – like the system proposed in section 2 – could be much more attractive than a pure carbon tax. The next section sets up a model to analyze the optimal design of such a system.

4. A general equilibrium model with carbon leakage at the extensive and the intensive margin

According to conventional wisdom, an efficient climate policy requires a uniform carbon price throughout the economy to equalize the marginal abatement cost across sectors. The model set

[3] Beck et al. (2019) use a version of the GTAP general equilibrium model of the world economy to estimate rates of carbon leakage from the Danish economy. They find a leakage rate of roughly 50 percent for the economy as a whole. A large part of that leakage stems from the mechanics of the European Emissions Trading System which is not modelled in the present paper. The GTAP model does not distinguish explicitly between carbon leakage at the extensive and the intensive margin, but the simulation results from the model do not seem to contradict the finding here that, when leakage occurs via the relocation of firms, the leakage rate is likely to be much higher than the rate of leakage at the intensive margin.

up in this section serves to illustrate how this policy prescription changes in an open economy with a commitment to reduce domestic emissions when the government is also concerned about carbon leakage. The model focuses on the optimal differentiation of the carbon tax between the household and the business sector and on the optimal balance between the tax rate and the lump sum subsidy within the business sector. It should be straightforward to generalize the analysis to study the optimal carbon tax differentiation within the business sector, depending on the different risks of leakage from the various subsectors.

We consider a small open economy with a continuum of potentially mobile competitive firms that all have the same production function (3) but have different firm-specific costs of relocating their activity abroad. The final good produced by domestic firms is traded internationally at a world market price which we now take to be exogenous and normalize at unity. Firms producing at home are subject to the carbon tax scheme (2) which may be avoided by moving production abroad. Hence the maximum attainable profits at home and abroad are given by the profit functions

$$\pi = \pi\left(p+\tau_b\right), \quad \frac{d\pi}{d\left(p+\tau_b\right)} = -e, \quad \pi^* = \pi\left(p\right), \quad \frac{d\pi^*}{dp} = -e^*, \quad \pi < \pi^* \text{ for } \tau_b > 0, \tag{19}$$

where the energy demand functions derived from profit maximization take the form[4]

$$e = e\left(p+\tau_b\right), \qquad e^* = e\left(p\right), \qquad e' < 0. \tag{20}$$

The government knows the distribution of the firm-specific international mobility costs (the costs of relocation) across the spectrum of firms, but it cannot observe the mobility cost of the individual firm, so it cannot differentiate the lump-sum subsidy T_b granted under the carbon tax scheme (2) according to a firm's degree of mobility.[5] In line with (7), a firm *i* with mobility cost c_i will prefer to remain located at home after the introduction of the carbon tax scheme if

[4] Note that since firms use the same technology at home and abroad, the functions $\pi(\bullet)$ and $e(\bullet)$ are the same across jurisdictions, but the energy cost is higher by the amount τ_b when a firm produces at home rather than abroad.
[5] Further, since all firms have the same production function (3), they all have the same historical emissions determining the baseline emission level $\overline{e}$ in (2). Hence they all receive the same subsidy T_b.

$$\pi(p)-\pi(p+\tau_b)-T_b \leq c_i. \tag{21}$$

Let $c(\tau_b, T_b)$ denote the mobility cost c_i that satisfies (21) with equality and let $F(c(\tau_b, T_b))$ denote the fraction of firms with a mobility cost less than or equal to $c(\tau_b, T_b)$. If mobility costs are distributed uniformly across firms over the interval from zero to $\bar{c} > 0$, we have $F(c(\tau_b, T_b)) = \frac{c(\tau_b, T_b)}{\bar{c}}$, where we assume that $c(\tau_b, T_b) < \bar{c}$.[6] We may then define the function

$$s(\tau_b, T_b) \equiv 1 - \frac{c(\tau_b, T_b)}{\bar{c}}, \tag{22}$$

indicating the fraction of firms that will stay at home after the introduction of the carbon tax scheme. From (19), (21), (22) and the definition of $c(\tau_b, T_b) < \bar{c}$ it follows that

$$\frac{\partial s}{\partial \tau_b} = -\frac{e}{\bar{c}} < 0, \qquad \frac{\partial s}{\partial T_b} = \frac{1}{\bar{c}} > 0. \tag{23}$$

Thus the government may reduce carbon leakage at the extensive margin by lowering the carbon tax rate on the business sector and/or by increasing the lump sum subsidy to domestic location, e.g., by increasing the baseline emission level $\bar{e}$ in (2).

A firm which is indifferent between producing at home or abroad has the mobility cost $c(\tau_b, T_b)$. The mobility costs for firms that decide to move out are uniformly distributed over the interval from zero to $c(\tau_b, T_b)$, so the average mobility cost for the firms that relocate in reaction to the carbon tax scheme is $\frac{1}{2}c(\tau_b, T_b) = \frac{\bar{c}}{2}\left[1 - s(\tau_b, T_b)\right]$, where the equality follows from (22). Without loss of generality, we may set the number of firms operating in the domestic economy before the introduction of the tax scheme equal to 1. These firms are assumed to be owned by domestic residents who retain their ownership if the firms move abroad.[7] The total net profits Π accruing

[6] Assuming a uniform distribution of mobility costs simplifies the exposition without undermining the validity of the optimal tax formulas derived below. The assumption $c(\tau_b, T_b) < \bar{c}$ ensures that some firms will always choose to stay at home because it is too costly for them to move abroad.

[7] The analysis could easily be generalized to account for an exogenous foreign ownership share of firms. This would not change our results in any important way.

to domestic residents after the introduction of the carbon tax scheme will then be given by the function

$$\Pi\left(\tau_b, T_b\right) = \overbrace{s\left(\tau_b, T_b\right)\left[\pi\left(p+\tau_b\right)+T_b\right]}^{\text{Profits from firms staying at home}} + \overbrace{\left[1-s\left(\tau_b, T_b\right)\right]\left\{\pi\left(p\right)-\underbrace{\tfrac{\bar{c}}{2}\left[1-s\left(\tau_b, T_b\right)\right]}_{\text{Average mobility cost}}\right\}}^{\text{Profits from firms relocating to foreign countries}}. \tag{24}$$

Using (19) and (22) through (24), we find that the function $\Pi\left(\tau_b, T_b\right)$ has the derivatives

$$\frac{\partial \Pi\left(\tau_b, T_b\right)}{\partial \tau_b} = -s\left(\tau_b, T_b\right)e < 0, \qquad \frac{\partial \Pi\left(\tau_b, T_b\right)}{\partial T_b} = s\left(\tau_b, T_b\right) > 0. \tag{25}$$

This completes the description of the business sector of the economy. The household sector is represented by a consumer who gains utility *U* from consumption of fossil energy (*E*) and consumption of the traded final good (*C*):

$$U = u\left(C, E\right), \quad \frac{\partial u}{\partial C} > 0, \quad \frac{\partial^2 u}{\left(\partial C\right)^2} < 0, \quad \frac{\partial u}{\partial E} > 0, \quad \frac{\partial^2 u}{\left(\partial E\right)^2} < 0. \tag{26}$$

The consumer's sources of income are the profits from firms and a government lump sum transfer T_h. Both types of income are taken as given by the individual consumer. The government levies a unit carbon tax at the rate τ_h on household energy consumption, so the consumer's budget constraint is

$$C + \left(p+\tau_h\right)E = \Pi\left(\tau_b, T_b\right) + T_h. \tag{27}$$

Maximization of the direct utility function (26) subject to the budget constraint (27) yields a household energy demand function of the form

$$E = E\left(P_h, Y\right), \quad \frac{\partial E}{\partial P_h} < 0, \quad \frac{\partial E}{\partial Y} > 0, \quad P_h \equiv p + \tau_h, \quad Y \equiv \Pi\left(\tau_b, T_b\right) + T_h, \tag{28}$$

and an indirect utility function $V\left(\bullet\right)$ with the properties

$$V = V\left(P_h, Y\right), \qquad \frac{\partial V}{\partial Y} = \lambda > 0, \qquad \frac{\partial V}{\partial P_h} = -\lambda E < 0, \tag{29}$$

where λ is the marginal utility of income (the Lagrange multiplier associated with the consumer budget constraint).

The government chooses its policy instruments with the purpose of attaining an exogenous target $\overline{\mathrm{E}}$ for CO_2 emissions from domestic territory, where $\overline{\mathrm{E}}$ is less than the laissez-faire level of emissions so that the target is binding. The focus on domestic emissions is in line with the principle underlying the United Nations Framework Convention on Climate Change according to which UN member states are responsible for emissions from their own territory. Thus we assume that the domestic government faces the climate policy constraint

$$\overbrace{s\left(\tau_b, T_b\right) e\left(p + \tau_b\right)}^{\text{Emissions from domestic firms}} + \overbrace{E\left(p + \tau_h, \Pi\left(\tau_b, T_b\right) + T_h\right)}^{\text{Emissions from domestic households}} = \overline{\mathrm{E}}. \tag{30}$$

The government must also respect its budget constraint which is

$$\tau_b s\left(\tau_b, T_b\right) e\left(p + \tau_b\right) + \tau_h E\left(p + \tau_h, \Pi\left(\tau_b, T_b\right) + T_h\right) = s\left(\tau_b, T_b\right) T_b + T_h. \tag{31}$$

The government is concerned about consumer welfare, but may also be concerned about the effect of its policy on the global climate via carbon leakage. Before the introduction of the carbon tax scheme domestic firms were emitting the amount $e\left(p\right)$ of CO_2. After the introduction of the tax the firms that remain at home will emit a total amount $s\left(\tau_b, T_b\right) e\left(p + \tau_b\right)$, so emissions from this group of firms fall by the amount $s\left(\tau_b, T_b\right)\left[e\left(p\right) - e\left(p + \tau_b\right)\right]$, while the firms that move abroad will emit a total amount $\left[1 - s\left(\tau_b, T_b\right)\right] e\left(p\right)$, representing an increase in emissions from foreign territory. The total carbon leakage from the business sector may thus be written as

$$\text{Carbon leakage from business sector} = \overbrace{\left[1 - s\left(\tau_b, T_b\right)\right] e\left(p\right)}^{\text{Leakage at the extensive margin}} + \overbrace{\rho_b s\left(\tau_b, T_b\right)\left[e\left(p\right) - e\left(p + \tau_b\right)\right]}^{\text{Leakage at the intensive margin}},$$

where the parameter $0 < \rho_b < 1$ is the rate of carbon leakage at the intensive margin. The last column in Table 1 shows that leakage at the intensive margin via the output market becomes quite small when the price elasticity of output demand is high, and in fact the model in section 3 implies that the leakage rate tends to zero as the elasticity of demand tends to infinity.[8] This might suggest that the leakage rate ρ_b should be set equal to zero in a small open economy facing an infinitely elastic demand for its output. However, in a general equilibrium setting carbon leakage may also occur via the international market for fossil fuel. Although a fall in domestic demand for fossil fuel in our small open economy would only have a negligible negative impact on the world fuel price, this tiny price effect would operate on a world demand that is huge relative to the domestic economy, so the resulting increase in foreign demand for fossil fuel could still be non-negligible relative to the fall in domestic demand. Indeed, Hoel (2012, pp. 87-88) shows that the leakage rate via the fossil fuel market in a small open economy is equal to $\varepsilon_e^d / \left(\varepsilon_e^s + \varepsilon_e^d\right)$, where ε_e^d is the numerical price elasticity of world demand for fossil fuel, and ε_e^s is the price elasticity of world supply. This justifies our assumption that $0 < \rho_b < 1$. Note that leakage via the fossil fuel market does not occur when a domestic firm relocates to a foreign country, since the firm will then continue to produce the same volume of output and demand the same amount of fossil fuel as it did domestically before the introduction of the carbon tax. This is why the rate of leakage at the extensive margin (occurring solely via the output market) is exactly equal to one in the expression above.

However, in the household sector some leakage could occur via the international market for fossil fuel as household demand for fossil fuel decreases in response to the carbon tax.[9] We therefore allow for the possibility of a positive rate of carbon leakage ρ_h from the household sector. In the laissez-faire regime before the introduction of the carbon tax scheme, total household emissions are $E\left(p, \pi\left(p\right)\right)$, since consumer income only consists of domestic profits under laissez faire where $T_h = 0$. After the tax scheme is implemented, households emit a total of $E\left(p + \tau_h, \Pi\left(\tau_b, T_b\right) + T_h\right)$ units of CO_2. We thus have

[8] This result also holds in the partial equilibrium model of leakage via the output market in Hoel (2012, pp. 88-89) which is specified in a different way than our model in section 3.

[9] In a member state of the European Union a fall in domestic demand for fossil-fuel based electricity would also generate a more direct leakage effect via the European Emissions Trading System.

$$\text{Carbon leakage from household sector} = \rho_h\left[E\left(p,\pi\left(p\right)\right)-E\left(p+\tau_h,\Pi\left(\tau_b,T_b\right)+T_h\right)\right].$$

Reflecting the government's concern about the (non-environmental) welfare of domestic consumers as well as its concern about carbon leakage, I assume that the government seeks to maximize a social welfare function of the form

$$\begin{aligned} SW = & V\left(p+\tau_h,\Pi\left(\tau_b,T_b\right)+T_h\right)-\eta\left[1-s\left(\tau_b,T_b\right)\right]e\left(p\right)-\alpha\rho_b s\left(\tau_b,T_b\right)\left[e\left(p\right)-e\left(p+\tau_b\right)\right] \\ & -\alpha\rho_h\left[E\left(p,\pi\left(p\right)\right)-E\left(p+\tau_h,\Pi\left(\tau_b,T_b\right)+T_h\right)\right], \quad 0\le\alpha\le\eta, \quad 0\le\rho_h<\rho_b<1, \end{aligned} \tag{32}$$

where the parameter η captures the strength of the government's aversion to carbon leakage at the extensive margin, and α reflects its aversion to leakage from business and household activities that remain located in the domestic economy. By assuming $\alpha \le \eta$, I allow for the possibility that leakage at the extensive margin may be considered more harmful than leakage at the intensive margin. For example, the social cost of adjusting to a new allocation of resources (not explicitly modelled here) could be larger for the local communities involved when some firms close down domestic operations completely in order to move abroad, compared to a situation where the same emission reduction is achieved via minor emission cuts across a large number of firms that remain located at home. Furthermore, it seems realistic that, on average, the rate of leakage from the household sector is lower than the leakage rate at the intensive margin of the business sector and that both leakage rates are less than 1, as assumed in (32).[10]

The government's policy problem is to choose the policy instruments τ_b, τ_h, T_b, and T_h so as to maximize the social welfare function (32) subject to the climate policy constraint (30) and the government budget constraint (31). The next section presents the solution to this problem.

[10] According to the survey by Carbone and Rivers (2017), simulations with computable general equilibrium models typically imply macroeconomic leakage rates between 10 and 30 percent. Beck et al. (2019) estimate a macroeconomic leakage rate for Denmark of 52 percent once one accounts for the mechanics of the European Emissions Trading System. For the Danish household sector they estimate a leakage rate of only 14 percent. Hence it seems safe to assume that $0<\rho_h<\rho_b<1$ in the context of our model.

5. The optimal carbon tax scheme

The solution to the optimal tax problem stated above is derived in Appendix B which shows that the optimal values of the parameters of the carbon tax scheme must satisfy the following conditions:[11]

$$\tau_h = SCC^d - \rho_h SCC^f, \tag{33}$$

$$\tau_b = SCC^d - \rho_b SCC^f, \tag{34}$$

$$T_b = \left(SCC_e^f - \rho_b SCC^f\right)e\left(p\right), \tag{35}$$

where

$$SCC^d \equiv \frac{\omega}{\lambda}, \qquad SCC^f \equiv \frac{\alpha}{\lambda}, \qquad SCC_e^f \equiv \frac{\eta}{\lambda}. \tag{36}$$

The variable ω is the shadow price associated with the climate policy constraint (30), measured in utility terms, so $SCC^d \equiv \omega / \lambda$ is the marginal social cost of meeting the target for reduction of domestic emissions, measured in units of the final consumption good. By analogy, SCC^f is the marginal social cost of carbon leakage from domestic economic activity (at the intensive margin), and SCC_e^f is the marginal social cost of carbon leakage at the extensive margin.

Before discussing the general solution to the optimal tax problem, it is instructive to consider some benchmark cases. First, suppose the government is solely concerned about emissions from domestic territory without any regard for carbon leakage. We then have $\alpha = \eta = 0$ in which case it follows from (36) that the optimal tax rules (33) through (35) boil down to the following:

$$\alpha = \eta = 0 \quad \Rightarrow \quad \tau_h = \tau_b = SCC^d, \qquad T_b = 0. \tag{38}$$

[11] The transfer T_h adjusts to satisfy the government budget constraint, given the carbon tax parameters satisfying (33) through (35).

This is the standard policy prescription of a uniform carbon tax rate across sectors to ensure a cost-effective attainment of a target for domestic CO_2 emissions. In this case there is no need for a lump sum subsidy to firms staying at home since this instrument can only affect carbon leakage at the extensive margin with which the government is not concerned, just as it does not worry about leakage at the intensive margin.

A polar case is one where the government is equally concerned about the country's contributions to foreign and domestic emissions, despite the fact that it may only be held responsible for domestic emissions under international climate treaties. In this case, where carbon leakage via the intensive as well as the extensive margin is considered just as harmful as domestic emissions, we have $\alpha = \eta = \omega,$ implying $SCC^d = SCC^f = SCC_e^f = SCC.$ Equations (33) through (36) then imply the following optimal tax policy:

$$\alpha = \eta = \omega \;\Rightarrow\; \tau_h = (1-\rho_h)SCC, \quad \tau_b = (1-\rho_b)SCC, \quad T_b = (1-\rho_b)e(p)SCC. \tag{39}$$

These results are highly intuitive. If emissions from the domestic household sector are allowed to increase by one unit, the resulting increase in global emissions will only be $1-\rho_h$ units, since the leakage to foreign countries will drop by ρ_h units. Hence the marginal social cost of a unit increase in domestic household emissions is only $(1-\rho_h)SCC$, so this is the optimal rate of carbon tax on household emissions. Similarly, when emissions from domestic firms go up by one unit, global emissions only increase by $1-\rho_b$ due to reduced leakage at the intensive margin, so the marginal social cost of emissions from domestic firms determining the optimal carbon tax rate is $(1-\rho_b)SCC,$ as stated in (39). The last equation in (39) says that the optimal lump sum subsidy T_b - which is effectively a premium for locating at home rather than abroad - is proportional to the difference between the leakage rate at the extensive margin (which is 1) and the leakage rate at the intensive margin (ρ_b), multiplied by the marginal social cost of carbon. Again, this is intuitive.

When the tax parameters τ_b and T_b take the values stated in (39), the government's net revenue from the carbon tax on firms will be $s\left[\tau_b e(p+\tau_b) - T_b\right] = s(1-\rho_b)\left[e(p+\tau_b) - e(p)\right]SCC$

which is negative, since $e\left(p+\tau_b\right) < e\left(p\right)$. In other words, when the government cares as much about foreign emissions as about domestic emissions, it will want to tax the household sector to be able to subsidize the domestic business sector via the lump sum grant T_b designed to counteract carbon leakage at the extensive margin.

The optimal tax rules in the general case laid out in equations (33) through (36) are also easy to interpret. For example, when a firm is allowed to emit an extra unit from domestic territory, the resulting marginal social cost seen in isolation is SCC^d, but this cost is partly offset by the marginal social gain $\rho_b SCC^f$ resulting from lower emissions abroad as carbon leakage at the intensive margin falls by the amount ρ_b. Hence the appropriate carbon tax rate om emissions from domestic firms is $SCC^d - \rho_b SCC^f$, as stated in (34). An analogous interpretation applies to the optimal carbon tax rate on household emissions specified in (33). Equation (35) says that the lump sum premium T_b for locating at home is proportional to the difference $SCC_e^f - \rho_b SCC^f$ between the marginal social cost of carbon leakage at the extensive and the intensive margin. Whether the net tax burden on the business sector is positive or negative will depend on the relative size of the marginal social costs SCC^d, SCC^f, and SCC_e^f.

Note that when carbon leakage does not occur via relocation of firms (which may generate additional welfare costs, as discussed earlier), there is no reason on utilitarian welfare grounds why the government should assign a higher marginal damage cost to foreign than to domestic emissions, since the impact on the global climate is the same. Hence we may reasonably assume that $SCC^f \leq SCC^d$. Moreover, we have argued that leakage via the extensive margin is at least as cosly as leakage via the intensive margin, implying $SCC_e^f \geq SCC^f$. Since theory and evidence suggests that the leakage rates ρ_h and ρ_b are less than one, it then follows from (33) through (35) that the optimal carbon tax rates as well as the lump sum subsidy for locating at home are all positive, as long as the government assigns a positive social cost to carbon leakage. On the plausible assumption that the rate of carbon leakage from the household sector is lower than the (average) leakage rate from firms operating on domestic territory, it also follows that firms should face a lower rate of carbon tax than households, since (33) and (34) imply that

$$\tau_b = \tau_h - SCC^f \cdot \left(\rho_b - \rho_h\right). \tag{40}$$

6. Concluding remarks

This paper has highlighted the importance of distinguishing between carbon leakage at the extensive and the intensive margin when designing an optimal carbon tax scheme. Carbon leakage at the extensive margin occurs when firms relocate their production plants from the domestic to a foreign country in response to a domestic carbon tax, while leakage at the intensive margin occurs when firms located in the domestic country lose world market shares to foreign competitors as a result of the domestic carbon tax. Our analysis showed that when leakage occurs at the extensive margin, the leakage rate – defined as the increase in foreign emissions relative to the decrease in domestic emissions – will most likely be much larger than the rate of leakage occurring at the intensive margin.

The most interesting and novel result in this paper is the finding that, when a society cares about the effects of its climate policy on foreign as well as domestic emissions, the optimal carbon tax scheme includes a lump sum subsidy to firms operating in the domestic economy, since such a subsidy reduces carbon leakage at the extensive margin. In practice such a subsidy could be implemented through a carbon tax scheme that taxes emissions above a historical baseline level and subsidizes emission reductions below the baseline level at a similar rate. Apart from mitigating carbon leakage, such a scheme has the advantage of allowing policy makers to ensure a more equitable distribution of the net cost of emission reductions across employees and business owners in different industries.

According to our analysis, the standard prescription of a uniform carbon tax across sectors applies only in the case where the government does not care at all about carbon leakage. In this case there is no rationale for a lump sum subsidy to firms locating at home. However, when the government assigns some social cost to leakage, even if it is less than the social cost assigned to domestic emissions, carbon tax rates should be differentiated across sectors according to their different leakage rates, and some amount of lump sum premium for locating firms at home rather than abroad becomes optimal to mitigate leakage at the extensive margin.

These optimal tax results were derived from a simple model of a small open economy facing fixed international terms of trade, but the qualitative conclusions regarding the optimality of differentiating carbon tax rates across sectors and offering a lump sum premium for locating at home should carry over to a setting with endogenous terms of trade. This conjecture is supported by the partial equilibrium analysis in section 3 which allowed for changes in relative world market prices[12] and showed that the difference between the rate of carbon leakage at the extensive and the intensive margin is in fact higher the lower the price elasticity of world demand for the good considered. This suggests that governments may be more eager to avoid carbon leakage at the extensive margin in a setting with low price elasticities where terms of trade effects are likely to be strong.

For simplicity the general equilibrium model in sections 4 and 5 assumed that firms do not differ in their energy intensity but only with respect to their mobility costs. However, the analysis in sections 3 through 5 strongly suggests that business sectors differing significantly in their energy intensity (represented by the $\beta-$parameter in the model of section 3) will have different rates of carbon leakage at the intensive margin and should therefore face different rates of carbon tax. On the other hand, a carbon tax system with a large number of different tax rates may be difficult to administer and may invite lobbyism by interest groups, so in practice policy makers may have to strike a balance between these concerns and concerns about allocative efficiency in abatement efforts.

One limitation of this study is that it neglects the interaction between the carbon tax and existing non-environmental market distortions due, for example, to other pre-existing taxes.[13] However, Kaplow (2004, 2013) argues that when the government can flexibly adjust a non-linear income tax schedule to strike an optimal balance between equity and efficiency, the marginal cost of public funds does in fact become equal to one, as assumed in this paper. In any case, it seems

[12] Recall that the output price P and the price ratio P/p were endogenous in the analysis in section 3.

[13] See, e.g., Bovenberg and Goulder (1996), Parry (1997), and Goulder (2013) for analyses of such interactions.

likely that the mechanisms highlighted by the present analysis will remain important for optimal carbon tax design in a more realistic model of the economy that allows for other market distortions.

APPENDIX A
PARTIAL EQUILIBRIUM ANALYSIS OF CARBON LEAKAGE

This appendix documents the results reported in equations (14) through (17) in section 3. We start by inserting the solutions for the output levels x and x^* stated in (9) and (10) and the demand function (11) into the condition (12) for market equilibrium when the domestic firm does not relocate. Using the definition of the ad valorem carbon tax rate $t \equiv \tau_b / p,$ this operation yields the following condition implicitly determining the equilibrium output price:

$$A^{\frac{1}{1-\beta}} \left(\frac{\beta P}{p} \right)^{\frac{\beta}{1-\beta}} \left[\left(\frac{1}{1+t} \right)^{\frac{\beta}{1-\beta}} + n \right] = aP^{-\varepsilon}. \quad \text{(A.1)}$$

Solving (A.1) for P yields the

Equilibrium price without relocation: $$P = \left\{ \frac{A}{a} \left(\frac{\beta A}{p} \right)^{\frac{\beta}{1-\beta}} \left[\left(\frac{1}{1+t} \right)^{\frac{\beta}{1-\beta}} + n \right] \right\}^{-\left(\frac{1-\beta}{\varepsilon(1-\beta)+\beta} \right)}. \quad \text{(A.2)}$$

According to (9), the emissions from a firm producing at home can be written as follows, using the definition of the ad valorem carbon tax rate:

Emissions from domestic location: $$e = \left(\frac{\beta AP}{p(1+t)} \right)^{\frac{1}{1-\beta}}. \quad \text{(A.3)}$$

Substituting (A.3) in (A.2) and rearranging, we obtain (14). The global emissions when the domestically-owned firm stays at home are

Global emissions without relocation: $$E^g = e + ne^*, \quad \text{(A4)}$$

where e is given by (14), and e^* is given by the first equation in (10). Inserting those relationships in (A.4) and collecting terms, one ends up with (15).

Consider now the case where the domestic firm relocates to a foreign country to avoid the domestic carbon tax. The output of the domestically owned and all foreign firms is now given by the second equation in (10). Substituting this equation along with the demand function (11) into the market equilibrium condition (13), we get

$$(1+n)A^{\frac{1}{1-\beta}}\left(\frac{\beta P}{p}\right)^{\frac{\beta}{1-\beta}} = aP^{-\varepsilon}, \tag{A.5}$$

which may be solved for P to give the

$$\text{Equilibrium price with relocation:} \qquad P = \left\{\frac{A}{a}\left(\frac{\beta A}{p}\right)^{\frac{\beta}{1-\beta}}(1+n)\right\}^{-\left(\frac{1-\beta}{\varepsilon(1-\beta)+\beta}\right)}. \tag{A.6}$$

When (A.6) is inserted in the first equation in (10), we obtain (16), and when (16) is inserted in the equation

$$\text{Total emissions with relocation:} \qquad E^g = (1+n)e^*, \tag{A.7}$$

we get (17).

APPENDIX B

DERIVING THE OPTIMAL CARBON TAX SCHEME

Using (30) through (32), we get the following Lagrangian for the optimal tax problem stated at the end of section 4,

$$L = V\left(p+\tau_h, \Pi\left(\tau_b, T_b\right)+T_h\right) - \eta\left[1-s\left(\tau_b, T_b\right)\right]e\left(p\right) - \alpha\rho_b s\left(\tau_b, T_b\right)\left[e\left(p\right)-e\left(p+\tau_b\right)\right]$$

$$-\alpha\rho_h\left[E\left(p, \pi\left(p\right)\right) - E\left(p+\tau_h, \Pi\left(\tau_b, T_b\right)+T_h\right)\right]$$

$$+\mu\left\{\tau_b s\left(\tau_b, T_b\right)e\left(p+\tau_b\right) + \tau_h E\left(p+\tau_h, \Pi\left(\tau_b, T_b\right)+T_h\right) - s\left(\tau_b, T_b\right)T_b - T_h\right\}$$

$$+\omega\left\{\bar{\mathrm{E}} - s\left(\tau_b, T_b\right)e\left(p+\tau_b\right) - E\left(p+\tau_h, \Pi\left(\tau_b, T_b\right)+T_h\right)\right\},$$

where the shadow price μ associated with the government budget constraint indicates the marginal cost of public funds, and the shadow price ω associated with the climate policy constraint indicates the marginal social cost of domestic emissions, with both shadow prices measured in utility terms. From this Lagrangian and the properties of the profit function and the indirect utility function stated in (25) and (29), respectively, we obtain the following first-order conditions for the optimal choice of policy instruments, where we use the simplifying notation $P_h \equiv p+\tau_h$ and $P_b \equiv p+\tau_b$:

$$\begin{gathered}\partial L/\partial T_h = 0 \quad \Rightarrow \\ \lambda + \mu\left(\tau_h \frac{\partial E}{\partial Y} - 1\right) - \left(\omega - \alpha\rho_h\right)\frac{\partial E}{\partial Y} = 0,\end{gathered} \tag{B.1}$$

$$\begin{gathered}\partial L/\partial \tau_h = 0 \quad \Rightarrow \\ -\lambda E + \mu\left(E + \tau_h \frac{\partial E}{\partial P_h}\right) - \left(\omega - \alpha\rho_h\right)\frac{\partial E}{\partial P_h} = 0,\end{gathered} \tag{B.2}$$

$$\begin{gathered}\partial L/\partial T_b = 0 \quad \Rightarrow \\ \lambda s + \mu\left(\tau_b e\frac{\partial s}{\partial T_b} + \tau_h s\frac{\partial E}{\partial Y} - s - T_b\frac{\partial s}{\partial T_b}\right) - \omega\left(e\frac{\partial s}{\partial T_b} + s\frac{\partial E}{\partial Y}\right) \\ +\eta e\left(p\right)\frac{\partial s}{\partial T_b} - \alpha\rho_b\left[e\left(p\right) - e\left(p+\tau_b\right)\right]\frac{\partial s}{\partial T_b} + \alpha\rho_h s\frac{\partial E}{\partial Y} = 0,\end{gathered} \tag{B.3}$$

$$\partial L / \partial \tau_b = 0 \quad \Rightarrow$$

$$-\lambda se + \mu\left(se + \tau_b e \frac{\partial s}{\partial \tau_b} + \tau_b s \frac{\partial e}{\partial P_b} - \tau_h se \frac{\partial E}{\partial Y} - T_b \frac{\partial s}{\partial \tau_b} \right) - \omega\left(e \frac{\partial s}{\partial \tau_b} + s \frac{\partial e}{\partial P_b} - se \frac{\partial E}{\partial Y} \right) \qquad \text{(B.4)}$$
$$+\eta e(p) \frac{\partial s}{\partial \tau_b} - \alpha\rho_b \left[e(p) - e(p+\tau_b) \right] \frac{\partial s}{\partial \tau_b} + \alpha\rho_b s \frac{\partial e}{\partial P_b} - \alpha\rho_h se \frac{\partial E}{\partial Y} = 0.$$

Since the government can use the lump sum household transfer T_h to balance its budget, the marginal cost of public funds (μ) is equal to the private marginal utility of income (λ).[14] Inserting $\mu = \lambda$, (B.1) and (B.2) both simplify to

$$\tau_h = \frac{\omega - \alpha\rho_h}{\lambda}. \qquad \text{(B.5)}$$

Solving (B.5) for ω and inserting the resulting expression along with $\mu = \lambda$ into (B.3) and (B.4), these first-order conditions boil down to

$$T_b = \left[\tau_b - \tau_h + \frac{\alpha}{\lambda}(\rho_b - \rho_h) \right] e + \left(\frac{\eta}{\lambda} - \frac{\alpha}{\lambda}\rho_b \right) e(p), \qquad \text{(B.6)}$$

$$T_b \frac{\partial s}{\partial \tau_b} = \left[\tau_b - \tau_h + \frac{\alpha}{\lambda}(\rho_b - \rho_h) \right] \left(e \frac{\partial s}{\partial \tau_b} + s \frac{\partial e}{\partial P_b} \right) + \left(\frac{\eta}{\lambda} - \frac{\alpha}{\lambda}\rho_b \right) e(p) \frac{\partial s}{\partial \tau_b}. \qquad \text{(B.7)}$$

Multiplying by $\partial s / \partial \tau_b$ on both sides of (B.6) and subtracting the resulting expression from (B.7), we find that

$$\tau_b - \tau_h = \frac{\alpha}{\lambda}(\rho_h - \rho_b), \qquad \text{(B.8)}$$

which may be inserted in (B.6) to give

$$T_b = \left(\frac{\eta}{\lambda} - \frac{\alpha}{\lambda}\rho_b \right) e(p). \qquad \text{(B.9)}$$

Finally, we may insert (B.5) in (B.8) to obtain

$$\tau_b = \frac{\omega - \alpha\rho_b}{\lambda}. \qquad \text{(B.10)}$$

[14] This result becomes immediately clear if one uses the government budget constraint (31) to eliminate T_h from the Lagrangian in which case μ drops out and the first-order conditions become identical to (B.5) through (B.7).

Given the definitions of SCC^d, SCC^f and SCC_e^f stated in (36), we see that (B.5) is identical to (33), (B.9) is the same as (35), and (B.10) is identical to (34).

APPENDIX C

ACCOUNTING FOR CARBON LEAKAGE

VIA CHANGES IN NET EXPORTS OF FINAL GOODS

The analysis above focuses on the carbon leakage that occurs as the domestic carbon tax reduces the use of fossil fuel in the domestic economy. However, in general equilibrium the carbon tax will also affect foreign emissions by changing the net exports of final goods from the domestic economy. Specifically, the carbon tax scheme reduces the import of fossil fuel and increases profit income from abroad by inducing some domestically owned firms to relocate to foreign countries. To maintain current account balance, the domestic net export of final goods must therefore go down, but since the world demand for these goods is unchanged (given the fixed world market price), the lower domestic supply must be offset by a corresponding increase in foreign supply which requires an increase in foreign fossil fuel use as foreign firms increase their production. For example, if the production function takes the form $f\left(e\right)=Ae^{\beta}$, if the price of the final traded good is 1, and the relative price of fossil fuel is *p*, and if the burning of one unit of fossil fuel emits one unit of CO_2, it is easy to show that when competitive firms maximize their profit, the emission per unit of final goods produced abroad will be

$$\rho_c \equiv \frac{e}{f} = \frac{\beta}{p}. \tag{C.1}$$

This appendix shows how carbon leakage via the net export channel may be integrated in our analysis of optimal carbon taxation. Adding the household budget constraint (27) and the government budget constraint (31), using (24) and the definition of profits $\pi \equiv f\left(e\right)-\left(p+\tau_b\right)e$, and collecting terms, we obtain the economy's aggregate resource constraint which requires the country's current account to balance:

$$\underbrace{\overbrace{f\left(e\left(p+\tau_b\right)\right)-C}^{\text{Value of net export of final goods}}+\overbrace{y^f\left(\tau_b,T_b\right)}^{\text{Net profit income from abroad}}-\overbrace{p\left[s\left(\tau_b,T_b\right)e\left(p+\tau_b\right)+E\left(p+\tau_h,\Pi\left(\tau_b,T_b\right)+T_h\right)\right]}^{\text{Value of total import of fossil fuel to firms and households}}}_{\text{Current account}}=0, \tag{C.2}$$

$$y^f\left(\tau_b,T_b\right)\equiv\overbrace{\left[1-s\left(\tau_b,T_b\right)\right]\left\{\pi\left(p\right)-\tfrac{\bar{c}}{2}\underbrace{\left[1-s\left(\tau_b,T_b\right)\right]}_{\text{Average mobility cost}}\right\}}^{\text{Profits from firms relocating to foreign countries}}.$$

From (21), (22), (23) and the definitions of the mobility cost function $c\left(\tau_b,T_b\right)$ and the profit function $y^f\left(\tau_b,T_b\right)$, one can show that

$$\frac{\partial y^f}{\partial\tau_b}=-\pi^f\frac{\partial s}{\partial\tau_b},\qquad\frac{\partial y^f}{\partial T_b}=-\pi^f\frac{\partial s}{\partial T_b},\qquad\pi^f\equiv\pi\left(p\right)-c\left(\tau_b,T_b\right), \tag{C.3}$$

where π^f is the net profit accruing to the "marginal" firm that decides to relocate to a foreign country in response to the domestic carbon tax scheme. Before the introduction of the carbon tax scheme there were no domestically-owned firms operating abroad, so the net export of final goods from the domestic economy were

$$NX_0\equiv f\left(e\left(p\right)\right)-C_0=e\left(p\right)+E\left(p,\pi\left(p\right)\right). \tag{C.4}$$

According to (C.1), (C.2), and (C.4) we can now write the carbon leakage L_c generated by the carbon tax scheme via the change in net exports as

$$L_c=\rho_c\left\{NX_0+\overbrace{y^f\left(\tau_b,T_b\right)-p\left[s\left(\tau_b,T_b\right)e\left(p+\tau_b\right)+E\left(p+\tau_h,\Pi\left(\tau_b,T_b\right)+T_h\right)\right]}^{=\,-\,\left(\text{net export of final goods after introduction of the carbon tax}\right)}\right\}. \tag{C.5}$$

For simplicity, assume that all forms of carbon leakage are considered to be equally harmful, and let α denote the social unit welfare cost of leakage, measured in utility terms. The social welfare function (32) then modifies to

$$SW = \overbrace{V\left(p+\tau_h, \Pi\left(\tau_b, T_b\right)+T_h\right)}^{\substack{\text{Non-environmental welfare} \\ \text{of representative household}}} - \alpha \overbrace{\left[1-s\left(\tau_b, T_b\right)\right]e\left(p\right)}^{\substack{\text{Leakage from the} \\ \text{extensive margin of firms}}} - \alpha \overbrace{\rho_b s\left(\tau_b, T_b\right)\left[e\left(p\right)-e\left(p+\tau_b\right)\right]}^{\text{Leakage from the intensive margin of firms}}$$

$$-\alpha \overbrace{\rho_h \left[E\left(p, \pi\left(p\right)\right) - E\left(p+\tau_h, \Pi\left(\tau_b, T_b\right)+T_h\right)\right]}^{\text{Leakage from reduced fossil fuel consumption in households}} \tag{C.6}$$

$$-\alpha \overbrace{\rho_c \left\{NX_0 + y^f\left(\tau_b, T_b\right) - p\left[s\left(\tau_b, T_b\right)e\left(p+\tau_b\right) + E\left(p+\tau_h, \Pi\left(\tau_b, T_b\right)+T_h\right)\right]\right\}}^{\text{Leakage from fall in net export of final goods}}.$$

Applying the Lagrangian technique described in Appendix B, we can derive the optimal carbon tax scheme by maximizing the social welfare function (C.6) with respect to the policy instruments τ_b, τ_h, T_b, and T_h, subject to the climate policy constraint (30) and the government budget constraint (31), using our results regarding the derivatives of the functions $s\left(\tau_b, T_b\right)$, $\Pi\left(\tau_b, T_b\right)$, and $y^f\left(\tau_b, T_b\right)$. This exercise yields the following expressions for the optimal values of the tax instruments:

$$\tau_h = SCC^d - \left(\rho_h + \rho_c\right)SCC^f, \tag{C.7}$$

$$\tau_b = SCC^d - \left(\rho_b + \rho_c\right)SCC^f, \tag{C.8}$$

$$T_b = \left[\left(1-\rho_b\right)e\left(p\right) + \rho_c \pi^f\right]SCC^f, \tag{C.9}$$

$$SCC^d \equiv \frac{\omega}{\lambda}, \qquad SCC^f \equiv \frac{\alpha}{\lambda}. \tag{C.10}$$

The variables SCC^d and SCC^f are the marginal social costs of emissions from domestic and foreign territory, respectively, measured in units of the final consumption good. The interpretation of the optimal tax formulas (C.7) through (C.9) is analogous to the interpretation of the corresponding formulas presented in section 5, but now the optimal policy also allows for carbon leakage via changes in the net export of the final good. Formula (C.9) is highly intuitive, since the term $\left[(1-\rho_b)e(p)+\rho_c\pi^f\right]$ is the additional leakage occurring when a firm relocates from the domestic to the foreign economy: the term $(1-\rho_b)e(p)$ is the difference between leakage at the extensive and the intensive margin (starting from an equilibrium without tax), and the term $\rho_c\pi^f$ is the leakage occurring as the additional net profit income from abroad allows a fall in net exports of the final good, given that the current account must remain in balance.

In summary, allowing for carbon leakage via the net export channel tends to reduce the optimal carbon tax rates on households and firms and to increase the lump sum subsidy for locating a firm at home rather than abroad.

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